DOCUMENT RESUME

ED 334 070 SE 052 149

TITLE Guidelines for Teaching Geometry.
INSTITUTION Texas Education Agency, Austin. Div. of Curriculum Development.
PUB DATE 90
NOTE 62p.
AVAILABLE FROM Texas Education Agency, 1701 North Congress Avenue, Austin, TX 78701 ($1.00).
PUB TYPE Guides - Classroom Use - Teaching Guides (For Teacher) (052)

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS *Geometry; High Schools; *Instructional Materials; Learning Activities; Learning Strategies; Mathematics Education; *Mathematics Instruction; *Secondary School Mathematics; State Curriculum Guides; Teaching Methods
IDENTIFIERS *Texas

ABSTRACT

This guidebook is designed to assist teachers and other local school district personnel in the planning and the teaching of the high school geometry course, with essential elements as mandated by the Texas State Board of Education in 1989. Research on student learning in geometry provides the logical rationale for the instructional sequencing and techniques presented in this course. The publication presents the philosophy and the intent of the geometry course and discusses teacher preparation, student credits and prerequisites, the required essential elements of instruction, and the integrated use of technology. Included are sample pupil performance objectives and classroom activities that illustrate how the essential elements of the geometry course can be taught. Local school district personnel are advised to use these guidelines, both suggested and stated, in the development of their own curriculum documents for a geometry course. Four appendices contain a description of the van Hiele levels of mental development, activities to help students achieve the levels, a geometry instructional plan, and 64 references. (JJK)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED334070

GUIDELINES FOR TEACHING GEOMETRY

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☑ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

Texas Education Agency
Austin, Texas

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

Linda Kemp

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

SE 052 149

BEST COPY AVAILABLE

GUIDELINES FOR TEACHING GEOMETRY

This publication is not copyrighted; any or all sections may be duplicated. After an initial free distribution to authorized institutions, additional copies may be purchased for $1.00 from the Publications Distributions Office.

Texas Education Agency
1701 North Congress Avenue
Austin, Texas 78701

FOREWORD

Guidelines for Teaching Geometry is designed to assist teachers and other school district personnel in the planning and teaching of the high school Geometry course. The publication presents the philosophy and intent of the course and discusses teacher preparation, credits and prerequisites, the required essential elements of instruction, and the use of technology. Also included are sample objectives and activities to illustrate how the essential elements for Geometry can be taught. School district personnel may want to use these suggestions to develop their own curriculum documents for Geometry.

We hope these guidelines will be useful to district personnel in planning and teaching Geometry and in equipping the Geometry classroom.

W.N. Kirby
Commissioner of Education

STATE BOARD OF EDUCATION

(State Board for Vocational Education)

MONTE HASIE, Lubbock
Chairman of the State Board of Education
District 15

CAROLYN HONEA CRAWFORD, Beaumont
Vice Chairman of the State Board of Education
District 7

MARY HELEN BERLANGA, Corpus Christi
Secretary of the State Board of Education
District 2

Board Members

BOB AIKIN, Commerce
District 9

RAY ALEXANDER, Houston
District 4

MARY HELEN BERLANGA, Corpus Christi
District 2

EMMETT J. CONRAD, Dallas
District 13

CAROLYN HONEA CRAWFORD, Beaumont
District 7

ROBERT CUMMINGS, Houston
District 6

WILL DAVIS, Austin
District 10

WILLIAM L. HUDSON, Wichita Falls
District 14

GERALDINE MILLER, Dallas
District 12

JANE NELSON, Lewisville
District 11

RENE NÚÑEZ, El Paso
District 1

MARY PERKINS, Lufkin
District 8

JOHN H. SHIELDS, San Antonio
District 5

ESTEBAN SOSA, San Antonio
District 3

W.N. KIRBY, Commissioner of Education
(Executive Officer of the State Board of Education)

Committees of the State Board of Education

PERMANENT SCHOOL FUND
ESTEBAN SOSA, Chairman
BOB AIKIN
MONTE HASIE
WILLIAM L. HUDSON
GERALDINE MILLER
RENE NUNEZ

SCHOOL FINANCE
WILL D. DAVIS, Chairman
BOB AIKIN
ROBERT D. CUMMINGS
WILLIAM L. HUDSON
ESTEBAN SOSA

PERSONNEL
CAROLYN HONEA CRAWFORD, Chairman
EMMETT J. CONRAD
RENE NÚNEZ
JOHN H. SHIELDS

LONG-RANGE PLANNING
EMMETT J. CONRAD, Chairman
ROBERT D. CUMMINGS
WILL D. DAVIS
JANE NELSON
MARY KNOTTS PERKINS
JOHN H. SHIELDS

STUDENTS
GERALDINE MILLER, Chairman
RAYMOND A. ALEXANDER
MARY HELEN BERLANGA
MONTE HASIE
JANE NELSON
MARY KNOTTS PERKINS

TEXAS EDUCATION AGENCY

This publication was developed by Agency staff members in the Office of the Deputy Commissioner for Curriculum and Professional Development.

VICTORIA BERGIN
Deputy Commissioner for Curriculum and Professional Development

GRACE GRIMES
Assistant Commissioner for Curriculum and Instruction

LEROY F. PSENCIK
Director, Division of General Education

Mathematics Section

CATHY SEELEY
Director of Mathematics

REBECCA BROOKS McCOY
Mathematics Specialist, Elementary

BARBARA MONTALTO
Mathematics Specialist, Middle School
and Junior High

BILLY L. HOPKINS
Mathematics Specialist, High School

PAT FOLMAR ROBINSON
Editor
Division of General Education

CONTENTS

Overview of the Course 1

Purpose and Philosophy 1

Credit and Prerequisite 2

Teacher Preparation 2

Staff Development 3

Essential Elements of Instruction, Objectives, and Activities 4

Essential Elements 4

Sample Objectives 7

Sample Activities 14

Evaluation 38

Local Evaluation 38

State Evaluation: Texas Assessment of Academic Skills 39

Appendixes 44

Appendix A: The van Hiele Levels of Mental Development in Geometry 44

Appendix B: Appropriate Activities to Help Students Move from One Level to Another 46

Appendix C: Geometry Instructional Plan 47

Appendix D: Resources 48

Acknowledgments 53

Overview of the Course

Purpose and Philosophy

Geometry provides experiences that help students develop understanding of shapes and their properties. It enables students to solve relevant problems and to apply geometric properties to real-world situations. The course builds on the conceptual foundation provided by the kindergarten-Grade 8 as well as the Algebra I essential elements of instruction in mathematics. The essential elements develop topics related to lines, segments, and angles; triangles; other polygons; circles; solid geometry; and measurement. Students are taught strategies for addressing problem situations in these topics through a synthetic approach, a transformational approach, a coordinate approach, and a vector approach. Development of student spatial skills is an important component of Geometry, and the course focuses on student use of these skills in everyday life applications. In addition, the teacher's and student's use of physical models, including computer graphic technology, helps develop student intuition and provides a basis for work with abstract concepts.

An emphasis of the course is on connections. For example, the interplay between geometry and algebra strengthens students' ability to formulate and analyze problems within and outside of mathematics. The course also stresses connections between various approaches within geometry. Thus, while students will at times work separately in synthetic, coordinate, and transformational geometry, teachers should provide opportunities throughout the course for students to translate and interact among these systems.

A major thrust of Geometry is the integration throughout the course of the use of current technology including calculators and computers. Students should use calculators and computers as problem-solving and discovery tools whenever possible. The required essential elements of Geometry are based on the assumption that students will have access to scientific calculators and that the teacher will have access to a computer for demonstration purposes. Computer graphic techniques may be used through computer or microcomputer software and other appropriate technology.

Students' development of proofs is an important component of Geometry but not the only focus of the course. The use of formal deductive proof is appropriate only as part of a sequential development, which includes concrete experiences, intuitive understandings, generalizations, reasoning activities, and formal proofs. Teachers may either reserve attention to formal deductive proofs for the second half of the course

or integrate them throughout the course using sequential development topic by topic. Either approach precludes a heavy emphasis on isolated formal proof during the first half of the geometry course. The phrase "make a convincing argument" used in the essential elements refers to students' communicating orally or in writing descriptions of their thoughts that have led to certain conclusions or generalizations; this process is intended as an informal preproof activity, stressing reasoning rather than form.

Research on student learning in geometry provides a logical rationale for the instructional sequence and techniques of the course. For example, teachers should consider the van Hiele research on developmental levels which can provide a basis for instruction. (For information on the van Hiele developmental levels, see Appendix A.)

Credit and Prerequisite

Students may receive one unit of credit for two semesters of Geometry. One unit of Geometry is required for the Advanced High School Program and for the Advanced High School Honors Program. Districts have the option of allowing students to receive one-half unit credit for one semester of Geometry. Districts also should set grading policies. State Board of Education rules allow districts to award grades by semester or to average grades for the first and second semesters.

The recommended prerequisite for Geometry is one unit of Algebra I. Although prerequisites are not included in State Board rules, the rules do require that districts develop placement policies for all high school courses which will ensure that students are placed at the appropriate academic levels. A prerequisite of Algebra I should ensure appropriate placement.

Teacher Preparation

Geometry must be taught by teachers who are certified in secondary mathematics. However, certification is a first step, not the ultimate step in the ongoing process of teacher preparation.

Teachers assigned Geometry must also keep up with the technological advances that impact teaching. Because of the many advances in technology, the teaching of Geometry, as well as other mathematics courses, is rapidly changing. For example, teachers should be committed to the regular use of hand-held calculators and computers in the Geometry classroom. Also, teachers now have access to graphing computer software enabling them to allow much greater student exploration and discovery than ever before. With students able to explore some topics in greater depth than previously, teachers must decide what topics need less emphasis. They must make decisions based on their continuous teacher preparation.

As part of their ongoing preparation, geometry teachers should stay abreast of developments in their field by reading professional journals and by attending mathematics conferences when possible. In these ways and others, effective teachers continually look for new and better ways to teach the topics of their course. They maintain high levels of motivation and strive to communicate to students the beauty of Geometry as well as its application to everyday life.

Staff Development

This publication reflects revisions that the State Board of Education made in the essential elements for Geometry in 1989. These revisions support a philosophy of teaching that incorporates the use of technology and an emphasis on the connections of Geometry to other mathematics curriculum areas. In many cases, teachers will not currently have the necessary training to teach Geometry as described in these guidelines, and extensive staff development will be necessary.

To help meet the needs of mathematics educators, the Texas Education Agency coordinated a Staff Development Project funded through EESA/Dwight D. Eisenhower grants. In the late 1980s, this project produced staff development modules in several mathematics subject areas, including high school geometry. The geometry module provides training for teachers in the use of technology in instruction, as well as in the use of instructional models and manipulatives to teach the revised Geometry curriculum. The three-day module training qualifies teachers for 18 hours of AAT credit. Teachers who take this staff development training will gain many new ideas and receive materials that will help them teach Geometry.

Information on training schedules for the geometry module is available from education service centers and the mathematics section of the Agency.

Essential Elements of Instruction Objective and Activities

Essential Elements

The State Board of Education in 1989 revised the essential elements of instruction for Geometry. School districts are to implement these revised essential elements by September 1990. The essential elements incorporate various approaches to the study of geometry including synthetic, coordinate, transformational, and vector. Synthetic geometry is the study of geometry by methods that focus on the relative positions of points, lines, and circles. Coordinate geometry is the study of geometry in which position is represented by coordinates. This approach to geometry mostly uses algebraic methods of reasoning. Transformational geometry is the study of operations on figures in a plane (e.g., reflections) that transform a pre-image of the figure to an image. This approach uses symmetry and properties of transformation in its reasoning. In a vector approach to geometry, two- and three-dimensional vectors are used, and deduction involves addition and multiplication of the vectors.

The revised Geometry essential elements are as follows:

1. Axiomatic systems. The student shall be provided opportunities to:

   (A) distinguish intuitively between the concepts of validity of an argument and truth of a statement;

   (B) distinguish between inductive and deductive reasoning;

   (C) use conditional statements in logical arguments;

   (D) investigate the relationship among a conditional statement and its converse, inverse, and contrapositive;

   (E) identify patterns of inference that produce valid conclusions and apply in real-world situations;

   (F) apply logical arguments to geometric problem situations; and

   (G) explore Euclidean geometry as an axiomatic system (undefined terms, defined terms, postulates, and theorems).

(2) Lines, segments, and angles. The student shall be provided opportunities to:

   (A) use a coordinate approach to:

(i) develop distance, midpoint, and betweenness properties;

(ii) investigate slopes of parallel and perpendicular lines using computer graphing techniques; and

(iii) write equations for lines given prescribed geometric conditions (locus).

(B) use a synthetic approach to:

(i) construct figures using a computer where appropriate;

(ii) make a convincing informal argument, orally or in writing, to justify angle relationships such as vertical, supplementary, complementary, congruent, and angles formed by transversals; and

(iii) write formal proofs related to parallel lines.

(C) use a vector approach to:

(i) determine whether two line segments are parallel; and

(ii) determine whether two line segments are perpendicular.

(D) solve real-world problems involving lines, segments, and angles.

(3) Triangles. The student shall be provided opportunities to:

(A) use a coordinate approach to:

(i) investigate specific examples of properties of segments associated with triangles (altitudes, medians, perpendicular bisectors, angle bisectors, etc.);

(ii) make generalizations inductively about these properties; and

(iii) make a convincing informal argument orally or in writing to validate these generalizations.

(B) use a transformational approach to:

(i) express translations, rotations, reflections, and dilations as functions; and

(ii) explore congruence and similarity through transformations.

(C) use a synthetic approach to:

(i) discover triangle congruence postulates through construction;

(ii) use triangle congruence propositions in proofs;

(iii) explore triangle similarity postulates and proportionality of associated linear measures (sides, altitudes, medians, angle bisectors, perimeters, etc.);

(iv) compare various proofs of the Pythagorean Theorem;

(v) investigate special right triangles;

(vi) develop trigonometric functions using the right triangle (sine, cosine, tangent);

(vii) use theorems related to angle measures of triangles in formal proofs; and

(viii) investigate triangle inequalities.

(D) solve real-world problems involving triangles.

(4) Other polygons. The student shall be provided opportunities to:

(A) use a coordinate approach to:

(i) explore specific examples of properties of quadrilaterals;

(ii) make generalizations inductively for these properties; and

(iii) make a convincing informal argument orally or in writing to validate these generalizations.

(B) use a transformational approach to:

(i) explore symmetry of polygons; and

(ii) generalize concepts of congruence and similarity.

(C) use a synthetic approach to:

(i) prove properties of quadrilaterals;

(ii) use properties of quadrilaterals in proofs;

(iii) explore specific examples of functional angle relationships in polygons, including regular polygons;

(iv) make generalizations inductively about these relationships; and

(v) make a convincing informal argument orally or in writing to validate these functional relationships.

(D) solve real-world problems involving polygons.

(5) Circles. The student shall be provided opportunities to:

(A) use a coordinate approach to graph equations and circles to:

(i) use the distance formula to develop the equations for circles; and

(ii) graph equations of circles.

(B) use a synthetic approach to:

(i) investigate lines, segments, and angles related to circles;

(ii) investigate inscribed and circumscribed circles using a computer; and

(iii) solve geometric problems involving segments and angles related to circles.

(C) solve real-world problems involving circles.

(6) Solid geometry. The student shall be provided opportunities to:

(A) use a synthetic approach to:

(i) investigate properties of polyhedra using concrete models (and computers where possible);

(ii) investigate properties of cylinders, cones, and spheres using concrete models (and computers where possible);

(iii) draw three-dimensional figures (using computers where possible); and

(iv) build three-dimensional models.

(B) use a transformational approach to explore symmetry of polyhedra and spheres in space.

(C) solve real-world problems involving three-dimensional figures.

(7) Measurement

(A) use a transformational approach to:

(i) explore distance and area as invariant conditions under translations, rotations, and reflections;

(ii) explore the ratio of distances as an invariant condition under dilations; and

(iii) explore angle measure as an invariant condition under translations, rotations, reflections, and dilations.

(B) use a synthetic approach to:

(i) make a convincing informal argument (using models and computers where possible) to validate area and volume formulas;

(ii) investigate the circumference and area of a circle as the limits of perimeters and areas of inscribed regular polygons;

(iii) use ratio to investigate arc length and areas of sectors of circles; and

(iv) investigate the proportional relationships among linear measures, areas, and volumes in similar figures.

(C) solve real-world problems involving measurement.

Sample Objectives

Following are 85 sample objectives that mathematics educators could use to teach the essential elements of Geometry. These objectives are not arranged in any particular instructional sequence; they are intended only to show examples of types of objectives teachers may use for each essential element. District educators may wish to use some or all of these objectives for their Geometry courses. If teachers use these sample objectives, they should arrange them in an instructional sequence compatible with district materials and teacher preference.

The numbers that follow the sample objectives refer to the required Chapter 75 essential elements and subelements to which these objectives are keyed. Districts must ensure that all the essential elements are taught.

ESSENTIAL ELEMENT 1: Axiomatic Systems

Reasoning

1. Recognize that circular reasoning, faulty analogies, conclusions based on false hypotheses, and arguments based on conditionals and their converses lead to invalid conclusions. [EE 1(E)]

2. Determine the characteristics of inductive and deductive reasoning, and identify the type of reasoning used to justify conclusions. [EE 1(B)]

Conditionals

3. Identify the hypothesis and conclusion for given conditional statements and write these statements in if-then form. [EE 1(C)]

4. Determine the equivalences of conditionals and contrapositives, and inverses and converses of the conditionals, using truth tables or Venn diagrams. [EE 1(D)]

Applications

5. Distinguish between logical arguments (or inferences) and statements of fact (e.g., advertisements, legal cases, games, etc.). [EE 1(A)]

6. Use truth tables or other inferential techniques (i.e., syllogistic reasoning, Law of Detachment, Venn diagrams, Law of Contrapositives, etc.) to establish the validity of arguments. [EE 1(E)]

7. Develop plans (oral or written) to arrive at given conclusions. [EE 1(F)]

8. Use indirect and deductive reasoning in problem situations. [EE 1(F)]

Axiomatic System Foundations

9. Differentiate among undefined terms, definitions, postulates, and theorems. [EE 1(G)]

10. Establish relationships among points, lines, and space and their subsets using appropriate symbols and terminology. [EE 1(G)]

ESSENTIAL ELEMENT 2: Lines, Segments, and Angles

Lines and Segments

11. Verify distance, midpoint, and betweenness properties using appropriate computer software, coordinate grids, dot paper, geoboards, and/or paper folding. [EE 2(A)(i)]

12. Using computer software, coordinate grids, and/or measurement instruments, demonstrate that distance is invariant under reflections, rotations, and translations. [EE 7(A)(i)]

13. Investigate slopes of parallel and perpendicular lines using computer software, coordinate grids, and/or plexiglass mirrors. [EE 2(A)(ii)]

14. Construct parallel and perpendicular lines using computer software, construction tools, paper folding, and/or plexiglass mirrors. [EE 2(B)(i)]

15. Verify postulates and theorems related to parallel and perpendicular lines using computer software. [EE 2(B)(ii)]

16. Write two-column and paragraph proofs related to parallel lines. [EE 2(B)(iii)]

17. Write equations for lines given locus definitions, graphs, or other prescribed geometric conditions. [EE (2)(A)(iii)]

18. Investigate the relationship between two vectors that are parallel or perpendicular using computer software, construction tools, coordinate grids, and/or algebraic methods. [EE 2(C)(i), EE 2(C)(ii)]

19. Solve real-world problems involving lines and segments. [EE 2(D)]

Angles

20. Demonstrate that angle measure is invariant under reflections, rotations, translations, and dilations using computer software, coordinate grids, and/or measurement instruments. [EE 7(A)(iii)]

21. Investigate angle measurement, angle addition, and angle bisection postulates using computer software, construction tools, measurement instruments, and/or plexiglass mirrors. [EE (2(B)(i)]

22. Make informal arguments, orally or in writing, to justify angle relationships such as relationships among vertical, supplementary, complementary, and congruent angles, and angles formed by transversals. [EE 2(B)(ii)]

23. Solve real-world problems involving angles. [EE 2(D))

ESSENTIAL ELEMENT 3: Triangles

Properties of Triangle Sides and Angles/Segments of a Triangle

24. Classify triangles according to lengths of sides and angle measures using computer software, coordinate grids, and/or measurement instruments. [EE 3(A)(i)]

25. Construct and investigate properties of special segments of triangles (medians, angle bisectors, altitudes, and perpendicular bisectors) using computer software, construction tools, paper folding, and/or plexiglass mirrors. [EE 3(A)(i)]

26. Establish points of concurrency for the special segments of triangles using computer software, construction tools, and/or measurement instruments. [EE 3(A)(ii)]

27. Generalize properties of special segments of triangles and give informal arguments to support these properties. [EE 3(A)(i), EE 3(A)(iii)]

28. Investigate and generalize triangle inequality relationships using computer software and/or measurement instruments. [EE 3(C)(vii)]

29. Write formal proofs (two-column or paragraph) using theorems relating to angle measures. [EE 3(C)(vii)]

Congruent Triangles

30. Use reflections, rotations, and translations to discover and verify congruence properties and correspondences for congruent figures. [EE 3(B)(i)]

31. Determine whether triangles are reflection-congruent or translation-congruent using computer software, coordinate grids, and/or plexiglass mirrors. [EE 3(B)(ii)]

32. Use transformation rules to plot images of triangles on coordinate grids. [EE 3(B)(ii)]

33. Write rules for isometries illustrated on coordinate grids. [EE 3(B)(i)]

34. Explore and establish triangle congruence postulates using computer software, construction tools, coordinate grids, and/or measurement instruments. [EE 3(C)(i)]

35. Apply the congruence properties and postulates in proofs and algebraic problems. [EE 3(C)(ii), 3(D)]

36. Solve real-world problems involving triangle congruence. [EE 3(D)]

Similar Triangles

37. Investigate and generalize properties of similar triangles using computer software, construction tools, coordinate grids, and/or measurement instruments. [EE 3(C)(iii), EE 7(B)(iv)]

38. Demonstrate that ratios of distances are invariant under dilations using computer software, coordinate grids, and/or measurement instruments. [EE 7(A)(ii)]

39. Investigate similarity of triangles through similarity transformations (composite of an isometry and dilation) using computer software, construction tools, and/or measurement instruments. [EE 3(B)(ii)]

40. Explore and establish the triangle similarity postulates and theorems using computer software, construction tools, coordinate grids, and/or measurement instruments. [EE 3(C)(iii)]

41. Investigate and generalize relationships of ratios of corresponding lengths (angle bisectors, altitudes, medians, and perimeters) in similar triangles using computer software, construction tools, and/or measurement instruments. [EE 3(C)(iii), EE 7(B)(iv)]

42. Apply the similarity properties and postulates in proofs and algebraic problems. [EE 3(C)(iii), EE 3(D)]

43. Solve real-world problems involving similar triangles. [EE 3(D)]

Right Triangles

44. Investigate and generalize relationships of the side lengths of right triangles (Pythagorean Theorem, Pythagorean triples, and geometric mean) using computer software, construction tools, coordinate grids, and/or measurement instruments. [EE 3(C)(v)]

45. Illustrate proofs of the Pythagorean Theorem using concrete models (geoboards, paper folding, and tiles) for the area and algebraic methods. [EE 3(C)(iv)]

46. Investigate and generalize relationships among lengths of sides in special right triangles (30°-60°-90°, 45°-45°-90°) using computer software, construction tools, coordinate grids, and/or measurement instruments. [EE 3(C)(v)]

47. Apply the Pythagorean Theorem and special right-triangle relationships in algebraic problems. [EE 3(C)(v)]

48. Solve real-world problems involving the Pythagorean Theorem. [EE 3(D)]

49. Investigate ratio relationships of right triangle side lengths for angle measures (sine, cosine, and tangent ratios) using measurement instruments and calculators or computer software. [EE 3(C)(vi)]

50. Solve indirect measurement problems using data gathered in field experiences. [EE 3(C)(viii)]

ESSENTIAL ELEMENT 4: Other Polygons

Properties

51. Investigate relationships among angles and sides of special quadrilaterals using computer software, coordinate grids, paper folding, and/or plexiglass mirrors. [EE 4(A)(i)]

52. Explore symmetries of quadrilaterals using computer software, coordinate grids, paper folding, and/or plexiglass mirrors. [EE 4(B)(i)]

53. Use transformation rules to plot images of quadrilaterals and to investigate symmetries of quadrilaterals. [EE 4(B)(i)]

54. Classify quadrilaterals according to their properties. [EE 4(A)(i)]

55. Generalize properties of special quadrilaterals and give informal and formal arguments to support these properties. [EE 4(A)(ii), 4(A)(iii), 4 (C)(i)]

56. Apply properties of special quadrilaterals in proofs and algebraic problems. [EE 4(C)(ii)]

57. Investigate and generalize concepts of congruence and similarity for quadrilaterals using computer software, coordinate grids, and/or plexiglass mirrors. [EE 4(B)(ii), EE 7(B)(iv)]

58. Investigate relationships among angles in polygons, including regular polygons, using computer software and/or measurement instruments. [EE 4(C)(iii)]

59. Generalize relationships among angles in polygons and give informal arguments to support these relationships. [EE 4(C)(iv), EE 4(C)(v)]

60. Solve real-world problems involving properties of quadrilaterals. [EE 4(D)]

Perimeter and Area

61. Develop area and perimeter formulas for triangles, squares, rectangles, parallelograms, rhombuses, trapezoids, and regular polygons using computer software, concrete models, and/or coordinate grids. [EE 7(B)(i)]

62. Give informal arguments to validate area and perimeter formulas. [EE 7(B)(i)]

63. Solve real-world problems involving perimeter and area of polygons. [EE 7(C)]

64. Investigate and generalize proportional relationships among linear measures, areas, and volumes in similar figures using computer software, coordinate grids, measurement tools, and/or algebraic methods. [EE 7(B)(iv)]

ESSENTIAL ELEMENT 5: Circles

Equations of Circles

65. Investigate second degree equations using graphing calculators, computer software, and/or coordinate grids, and identify the characteristics of those equations that are circles. [EE 5(A)(ii)]

66. Use the distance formula to develop a general equation for circles in a coordinate plane. [EE 5(A)(i)]

Relationships

67. Investigate relationships between angle and arc measures in circles using circular geoboards, computer software, and/or measurement instruments. [EE 5 (B)(i)]

68. Generalize relationships between angle and arc measures in circles and give informal arguments to support these relationships. [EE 5(B)(i)]

69. Apply relationships between angle and arc measures in proofs and algebraic problems. [EE 5(B)(iii), 5(C)]

70. Investigate relationships among segments and circles using computer software, coordinate grids, and/or measurement instruments. [EE 5(B)(i)]

71. Generalize relationships among angles and circles and give informal arguments to support these relationships. [EE 5(B)(i)]

72. Solve real-world problems involving angles, arc lengths, and line segments in circles. [EE 5(B)(iii), EE 5(C)]

Inscribed and Circumscribed Circles/Polygons

73. Investigate circles inscribed in and circumscribed about polygons using computer software and/or construction tools. [EE 5(B)(ii)]

74. Generalize information about the location of the centers of inscribed and circumscribed circles for triangles. [EE 5(B)(ii)]

Circumference, Area, and Arc Length

75. Develop formulas for circumference and area of circles, including limits, using computer software and/or concrete models. [EE 7(B)(ii)]

76. Develop methods for finding arc length and sector area in circles using computer software, concrete models, and/or paper folding. [EE 7(B)(iii)]

77. Solve real-world problems involving circumference, area, arc length, and sector area of circles. [EE 7(C)]

ESSENTIAL ELEMENT 6: Solid Geometry

Properties

78. Build polyhedron models using a variety of materials and investigate their properties (number of vertices, edges, and faces; shapes of faces). [EE 6(A)(i), EE 6(A)(iv)]

79. Investigate properties of cylinders, cones, and spheres using concrete models. [EE 6(A)(ii)]

80. Identify nets (fold-up patterns) for right prisms, pyramids, cones, and cylinders, and construct these models. [EE 6(A)(iv)]

81. Draw two-dimensional diagrams of three-dimensional figures. [EE 6(A)(iii)]

Surface Area and Volume

82. Determine surface area for three-dimensional figures using models and nets. [EE 7(B)(i)]

83. Give informal arguments that validate volume formulas for three-dimensional figures using models. [EE 7(B)(i)]

84. Investigate volume and surface area relationships among similar figures. [EE 7(B)(iv)]

85. Solve real-world problems involving three-dimensional figures. [EE 6(B), EE 7(C)]

Sample Activities

The following are sample activities that teachers could use for selected essential elements. The list is designed to illustrate methods of teaching and to help clarify what is meant by certain of the essential elements. The compilation is not meant to be a comprehensive list of activities to teach the essential elements, nor do these activities have to be used. However, the activities demonstrate the kind of work that students should be required to do in Geometry. The activities relate to the sample objectives on pages 7-14.

ESSENTIAL ELEMENT 3: Triangles

A. Activities for Sample Objectives 25 and 27

Objective 25
Construct and investigate properties of special segments of a triangle (medians, angle bisectors, altitudes, and perpendicular bisectors) using computer software, construction tools, paper folding, and/or plexiglass mirrors.

Objective 27
Generalize properties of special segments of triangles and give informal arguments to support these properties.

Subelements
3 (A)(i) and 3 (A)(ii)

Activity 1	Paper Folding

Task ► In this activity, students will construct the three medians of a triangle using paper folding to clarify the definition of a median.

Materials ► Unlined paper, scissors, straightedge

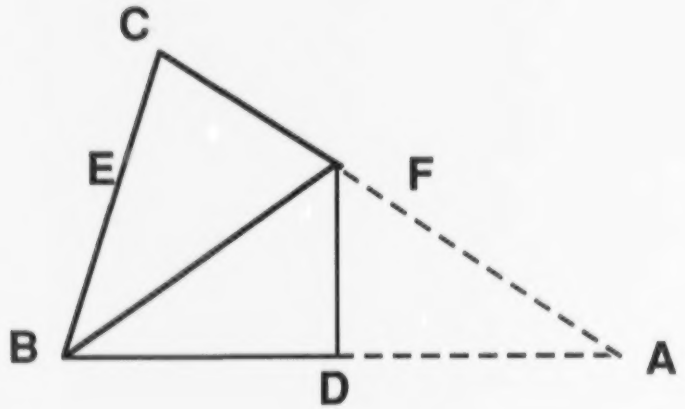

FIGURE 1

Procedures for Students ►

1. Draw and cut out a large, scalene, acute triangle ABC.
2. To find the midpoint of AB, fold point A onto point B and crease only on AB. Label this point D.
3. Repeat this procedure for BC and CA to locate the midpoints E and F, respectively.
4. Fold a crease from midpoint D to vertex C.
5. Repeat, creasing from the other midpoints to the opposite vertices. The segments formed are called medians.
6. Determine whether the three medians intersect at a common point.

Activity 2 — Reflection and Ratios

Task ► In this activity, students will investigate the relationship among the three medians using a plexiglass mirror and a straightedge.

Materials ► Tagboard or heavy paper, plexiglass mirror, scissors, straightedge, compass

Procedures for Students ►

1. Draw a large scalene acute triangle ABC.
2. Place the mirror across AB and reflect point B onto A. Mark midpoint D where the mirror's edge intersects AB.

3. Repeat this procedure, locating midpoints E and F on side BC and CA, respectively.
4. With a straightedge, draw the three medians by joining each midpoint with the opposite vertex.
5. Label the point where the medians intersect point G. This common point is called the point of concurrency.

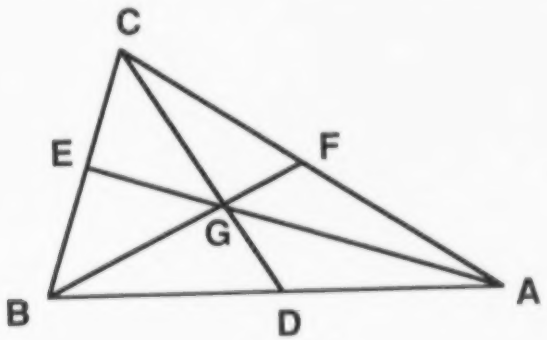

FIGURE 2

Investigation ►

1. Do the three medians appear to be congruent to each other?
2. Measuring with a compass, what are the longest and shortest medians?
3. What are the longest and shortest sides of triangle ABC?
4. The longest median is drawn to which side? The shortest median is drawn to which side? Compare your results with those of your classmates.
5. Is G the same distance from each vertex?
6. Is G the same distance from each side of triangle ABC?
7. Compare CG to GD. Is CG longer than GD? Estimate the ratio of CG:GD.
8. Now compare BG:GF and AG:GE. Are these ratios constant?
9. Write conjectures for the ratio of the segments of the medians of a triangle determined by the point of concurrency. Using a compass, test the conjectures about these ratios.

Activity 2 | **Optional Follow-Up**

Materials ► Stretchable ruler

FIGURE 3

Using a rubber band at least 3mm wide and 12 cm in circumference, cut it once. At equal intervals along the rubber band, mark four points. Put a thumbtack through each point.

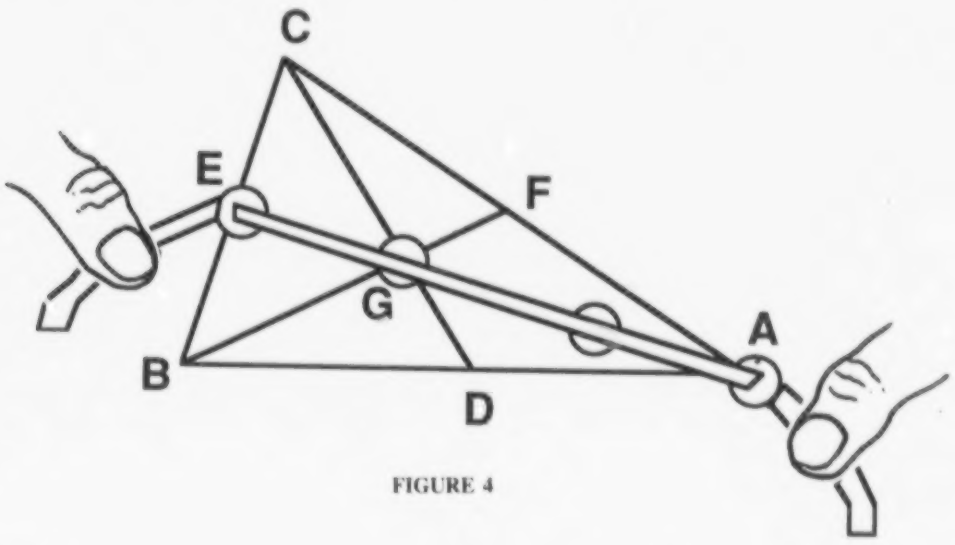

FIGURE 4

Procedures for Students ►

1. Use the large, scalene, acute triangle ABC with medians AE, BF, CD, and the point of concurrency G.
2. Grasp the stretchable ruler at the two outer tacks. Placing one tack on A and the other on E, determine if one of the other tacks falls on G. Make a comparison of the lengths AG and GE. Write a ratio for the comparison.
3. Using the stretchable ruler repeat the measuring procedure for medians BF and CD.
4. Write conjectures for the ratios of the segments of the medians of a triangle determined by the point of concurrency.

Extension ► Find a point on the cardboard triangle where it will balance on the tip of a pencil. Where is this point? The point is called the center of gravity of the triangle.

Activity 3	**Testing Conjectures With Computers**

Task ► In this activity, students will explore the relationship between the segments of a median of a triangle determined by the point of concurrency.

Materials ► Computer construction software, data/instruction sheet

Procedures for Students ►

1. Draw acute triangle ABC.
2. Draw the three medians AD, BE, and CF, and label the point of concurrency G.
3. Measure the segments of each median. Write the ratio of the lengths of these segments. Collect the data in the chart below.

Type of Triangle	AG	GD	AG:GD	BG	GE	BG:GE	CG	GF	CG:GF

4. Repeat this procedure for obtuse and right triangles.
5. Repeat the procedure on an isosceles triangle and an equilateral triangle.

Generalizations ► Study the data collected in the table, and write generalizations about the medians of a triangle.

B. Activities for Sample Objectives 24 and 28

Objective 24
Classify triangles according to lengths of sides and angle measures using computer software, coordinate grids, and/or measurement instruments.

Objective 28
Investigate and generalize triangle inequality relationships using computer software and/or measuring instruments.

Subelements
3 (A)(i) and 3 (C)(viii)

Activity 1	Measuring Angles and Sides

Task ► In this activity, students will measure sides and angles of triangles and make statements about their findings.

Materials ► Teacher-prepared drawings of triangles, ruler, protractor. The triangles to be measured should include scalene, isosceles, equilateral, acute, obtuse, right triangles, and combinations of these.

Procedures for Students ►

1. For each triangle, use a ruler to measure each of the sides of the triangles and record its measure on the side.
2. For each triangle, use a protractor to measure each of the angles of the triangles and record it in the angle.
3. Based on the measurements, make a list of statements about each triangle that seem true.

Example:

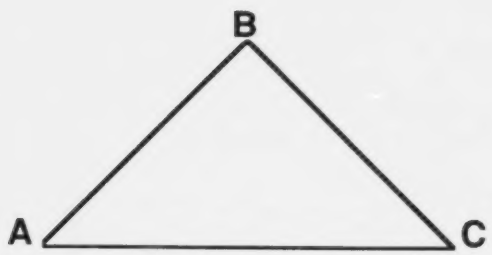

FIGURE 5

- Two sides have equal length.
- The base is the longest side of the triangle.
- Two of the angles are acute and one is obtuse.
- Two angles have equal measure.

Activity 2	Relationships of Measures of Angles and Lengths of Sides

Task ► In this activity, students will make a conjecture about the relationship between the measures of the angles and lengths of the sides.

Materials ► Six 20 x 20 grids, ruler, protractor, colored markers

Procedures for Students ►

1. Plot each set of points on a separate grid, and label the points. Use a straightedge to join the points to form a triangle.
   - A(4,8), B(6,-3), C(-4,0) [acute scalene]
   - D(0,8), E(6,-2), F(1,-5) [right scalene]
   - G(1,3), H(5,0), I(-2,-1) [acute isosceles]
   - J(-8,3), K(-2,8), L(4,3) [obtuse isosceles]
   - M(1,10), N(1,2), P(7,2) [right isosceles]
   - R(-5,-5), S(-2,-2), T(4,-4) [obtuse scalene]
2. Measure the sides of each triangle, and record the measure in the chart below. Then measure the angles, and record these measures.

Triangle Name	Side Measure	Angle Measure

Investigation ►

1. Identify each triangle by type (acute, right, obtuse, equilateral, isosceles, scalene).
2. Inspect each triangle. Color the longest side and color the largest angle. What pattern do you see?

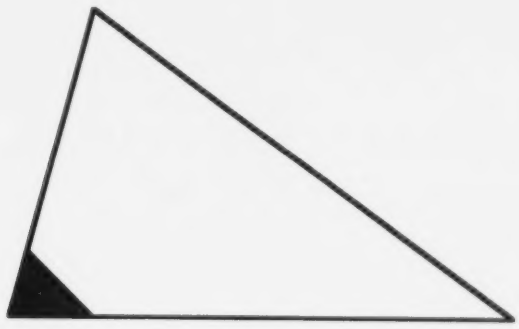

FIGURE 6

3. Use the measures in the chart to verify the pattern.
4. With a different color, mark the shortest side and the smallest angles of each triangle.
5. Make several conjectures about the relationship between the measures of the angles and the lengths of the sides.

Activity 3	Generalizing Using Computers

Task ► In this activity, students will make generalizations about the relationship between the measures of the angles and the lengths of the side opposite the angles.

Materials ► Computer construction software, data/instruction sheet

Procedures for Students ►

1. Construct triangle XYZ.
2. Measure the length of each side and the size of each angle. Record the measurements in a chart.
3. Repeat Steps 1 and 2 with a variety of triangles. Make sure to include triangles that have equal sides.

Generalizations ► Use the data collected to make generalizations about the relationship between the measures of the angles and the lengths of the sides of any triangle. Make generalizations about categories of triangles.

C. Activities for Sample Objectives 30 and 31

Objective 30
Use reflections, rotations, and translations to discover and verify congruence properties and correspondences for congruent figures.

Objective 31
Determine whether triangles are reflection-congruent or translation-congruent using computer software, coordinate grids, and/or plexiglass mirrors.

Subelements
3(B)(i) and 3(B)(ii)

Activity 1	Reflections

Task ► In this activity, students will match corresponding parts of reflected figures.

Materials ► Polygonal blocks, cut-out triangular shapes, plexiglass mirrors

Procedures for Students ►

1. Trace a block or a cut-out shape on paper, labeling the vertices.
2. Draw and label a line of reflection, and use the plexiglass mirror to reflect the figure about the line. Draw and label the reflection image.

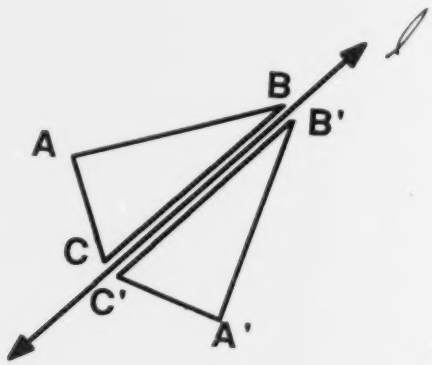

FIGURE 7

3. Place the original block or shape on the reflection to establish the congruence.
4. List the corresponding vertices.
5. Match the corresponding parts of the original figure and reflected image. This can be done either using the actual figure or by using the plexiglass mirror.
6. Repeat Steps 1 through 5 with different figures.

Activity 2	**Reflecting on the Coordinate Plane**

Task ► In this activity, students will reflect triangles across the *x*-axis and across the *y*-axis.

Materials ► Plexiglass mirrors, graph paper

Procedures for Students ►

1. Draw a scalene triangle on coordinate grid paper where the vertices A, B, and C of the triangle are in the first quadrant.

2. Use the plexiglass mirror to reflect triangle ABC across the x-axis, and label the corresponding vertices A′, B′, and C′.
3. Use the plexiglass mirror to reflect triangle ABC across the y-axis, and label the corresponding vertices A″, B″, and C″.
4. Complete the chart:

	Vertices		
Original triangle	A(,)	B(,)	C(,)
Reflection x-axis	A′(,)	B′(,)	C′(,)
Reflection y-axis	A′(,)	B′(,)	C′(,)

5. What relationships do you see between the original triangle and its reflections?
6. Repeat the above procedure for several triangles.

Investigation ►

1. Study all the figures reflected across the x-axis. What is true about these triangles? Are they congruent? Why? What seems to be true about the coordinates of the original triangle and those of its image?
2. What seems to be true about the triangles reflected across the y-axis?

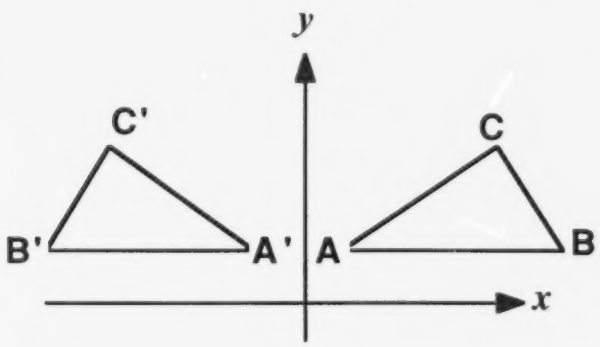

FIGURE 8

3. Draw AA′, BB′, and CC′. What appears to be true of the three line segments that connect the corresponding vertices?
4. What appears to be the relationship of each of these three line segments and the line of reflection?

5. Repeat Steps 3 and 4 using AA′, BB′, and CC′.
6. Make conjectures about reflecting triangles across an axis.

Activity 3	Using Computers for Reflections

Task ► In this activity, students will determine how the line of reflection is related to the pre-image and image.

Materials ► Computer software, data/information sheets

Procedures for Students ►

1. Draw line on the screen.
2. Draw a triangle ABC.
3. From each vertex, A, B, C, draw a line perpendicular to line and intersecting it at M, N, and P, respectively.
4. Extend AM to X so that AM = MX. Likewise, extend BN to Y and CP to Z so that BN = NY and CP = PZ.
5. Draw triangle XYZ. Measure the sides and angles of triangles ABC and XYZ. Record the information in the chart below.

Original Triangle		Reflected Image	
Side Lengths	**Angle Measures**	**Side Lengths**	**Angle Measures**

6. Repeat Steps 1 through 5 for several sets of triangles.

Generalizations ► Study all the data. What seems to be the relationship between the pre-image and the image triangles? What generalizations can you make?

Extensions ►

1. What happens to the image if triangle ABC has two of its sides cut by line ℓ? If one vertex is on line ℓ?
2. Use a coordinate grid to investigate the relationship of the coordinates of triangles and their images reflected across the line $y = x$.

ESSENTIAL ELEMENT 4: Other Polygons

A. Activities for Sample Objectives 54 and 55

Objective 54
Classify quadrilaterals according to their properties.

Objective 55
Generalize properties of special quadrilaterals and give informal and formal arguments to support those properties.

Subelements
4(A)(i), 4(A)(ii), 4(A)(iii) and 4(C)(i)

Activity 1	Matching Pairs

Task ► In this activity, students will compare pairs of quadrilaterals.

Materials ► Cards with the following figures drawn side-by-side: rectangle and square, square and rhombus, parallelogram and rhombus, parallelogram and rectangle. *(See Figure 9.)*

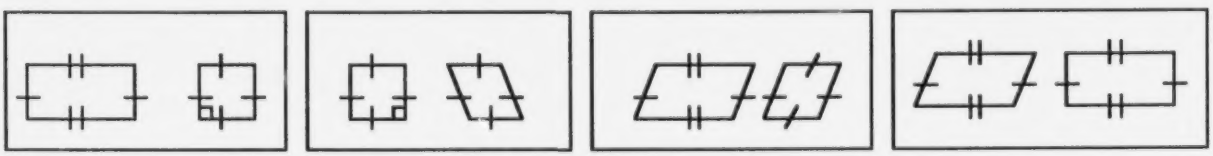

FIGURE 9

Procedure for Students ► For each of the pairs of shapes, give at least one way in which the pair is alike and one way in which they are different.

Activity 2 — Sorting Quadrilaterals

Task ► In this activity, students will write descriptions for sets of quadrilaterals.

Materials ► Two envelopes for each group of students. Envelope 1 will contain 12 parallelograms, some but not all of which are rectangles and squares; Envelope 2 will contain a rhombus that is not a square.

Procedures for Students ►

1. Sort the parallelograms in Envelope 1 into three sets and write a description of each set.
2. Open Envelope 2 and use the description of the sets to decide where the figure in that envelope belongs.
3. Describe the sets in a different way so that the figure from Envelope 2 belongs in another set.
4. Repeat Steps 1 through 3 sorting into two sets.

Investigation ►

1. How did you sort the figures into three (two) sets? Where does the figure from Envelope 2 belong?
2. Are there other ways to sort the figures into three (two) sets?
3. Complete this statement: "All _____s are _____s, but not all _____s are _____s."
4. Is there a way to combine a set of squares and a set of rhombuses and describe the set?
5. What statement can you make about squares and rhombuses?

Activity 3 — Properties of Special Quadrilaterals

Task ► In this activity, students will determine the properties of special quadrilaterals.

Materials ► Plexiglass mirrors; paper; scissors; sheets on which a rectangle, a square, a parallelogram, and a rhombus have been drawn

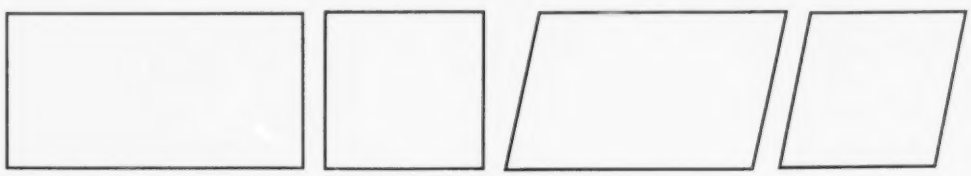

FIGURE 10

Procedures for Students ►

1. Determine which of the special quadrilaterals have the properties listed below using the method suggested.

Properties	Method	Parallelogram	Rhombus	Square	Rectangle
Opposite sides are parallel.	Mirror				
Opposite sides are congruent.	Paper folding				
Diagonals form congruent triangles.	Cut out & match				
Diagonals bisect each other.	Mirror				
Diagonals bisect opposite angles.	Mirror				
All sides are congruent.	Paper folding				
All angles are congruent.	Paper folding				
Diagonals are perpendicular.	Mirror				
Diagonals are congruent.	Cut out & match				
Opposite angles are congruent.	Cut out & match				

2. What patterns do you see?
3. Draw a Venn diagram that represents the relationship among the various special quadrilaterals.

Generalizations ► State generalizations about properties of quadrilaterals. Choose one of the generalizations and write a justification for it.

Extensions ►

1. A rectangle is frequently defined as a parallelogram with four right angles. Is it possible to leave out any of this definition and still have defined a rectangle? Is there another way?
2. Give at least two definitions for each of the other special quadrilaterals above. Be sure the definition is broad enough to cover all cases that belong (necessary conditions) and narrow enough to exclude those that do not belong (sufficient conditions).

ESSENTIAL ELEMENT 5: Circles

A. Activities for Sample Objectives 67 and 68

Objective 67
Investigate relationships between angle and arc measures in circles using circular geoboards, computer software, and/or measurement instruments.

Objective 68
Generalize relationships between angle and arc measures in circles and give informal arguments to support these relationships.

Subelement
5(B)(i)

Activity 1	Identifying Arcs and Angles in Circles

Task ► In this activity, students will develop the relationship between the measure of the central angle and the degree measure of its arc.

Materials ► Straightedge, compass, protractor, pencil

Procedures for Students ►

1. Draw a circle with center P and radius PR.
2. Draw a central angle with a measure of 60°. Label the second radius PS. Place a pencil along PR and rotate the pencil one complete rotation.

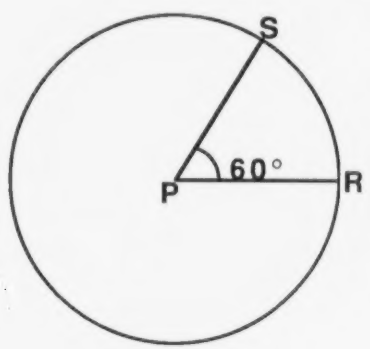

FIGURE 11

3. An arc is measured in arc degrees by the amount of rotation the arc describes. How many arc degrees are there in a complete rotation?
4. Measure minor arc RS by placing your compass on points R and S. How many times can you mark off this arc on circle P? Estimate the measure in arc degrees of minor arc RS.
5. How do the degree measure of RPS and the arc degree measure of RS compare?
6. Draw other circles and repeat Steps 3, 4, and 5 for measures of 90°, 72°, 45°, and 36°.

Activity 2	Measuring the Central and Inscribed Angle

Task ► In this activity, students will make a conjecture about the relationship between the degree measure of a central angle and the degree measure of an inscribed angle intercepting the same arc.

Materials ► Straightedge, compass, protractor

Procedures for Students ►

1. Draw circle O with radius between one and two inches.
2. Pick points R and S on circle O. Choose a point Z on the major arc RS and draw RZS. This angle is called an inscribed angle. Draw ROS. Which angle has the larger measure?
3. Draw circle O and choose points A and B on the circle. Draw the central angle and the inscribed angle determined by minor arc AB. Measure the central angle and record its measure on the figure. Measure the inscribed angle and record its measure.

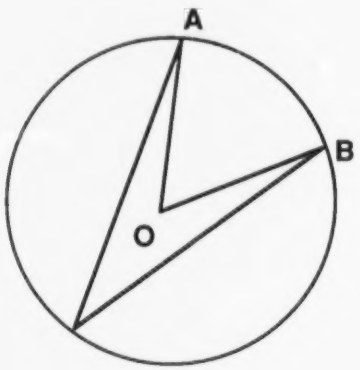

FIGURE 12

4. Repeat the procedures outlined in step 3 for several circles.
5. Write an equation that describes this relationship.

Activity 3	Test the Conjecture Using Measuring Instruments

Task ► In this activity, students generalize about the relationship between the degree measures of an inscribed angle and its intercepted arc.

Materials ► Compass, straightedge, protractor

Procedures for Students ►

1. Draw a circle. Draw an inscribed angle and the central angle that intercept the same arc. Measure both angles. Record the measures in the chart below.

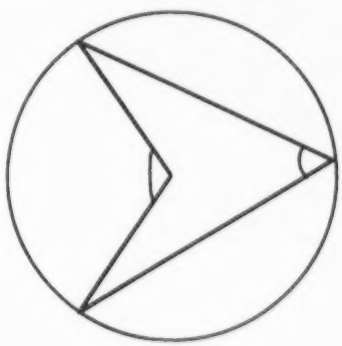

FIGURE 13

Measure of Central Angle	Measure of Intercepted Arc	Measure of Inscribed Angle

2. Repeat the procedure with other circles and record the data. Collect the data on a large chart on the classroom wall, the chalkboard, or an overhead projector.

Generalization ► Develop a generalization about the degree measure of the arc and the measure of the inscribed angle.

Extensions ►

1. Design an investigation for finding the measure of an angle when its vertex is in the interior of the circle.
2. Design an investigation for finding the measure of an angle when its vertex is in the exterior of the circle.

B. Activities for Sample Objective 76

Objective 76
Develop methods for finding arc length and sector area in circles using computer software, concrete models, and/or paper folding.

Subelement
7(B)(iii)

Activity 1	Draw and Cut

Task ► In this activity, students will draw and cut out sectors of circles and compare them for size.

Materials ► Scissors, compass, straightedge

Procedures for Students ►

1. Use a compass to draw a circle with a large radius with a center at O. Use a protractor to draw a central angle with a measure of 40°.
2. Extend the rays of the angle to intersect the circle at points A and B.
3. Cut out the region bounded by OA, OB, and arc AB. This is a sector of circle O.
4. Repeat Steps 1, 2, and 3 with three more circles of different radii.
5. Arrange the sectors cut from the four circles in order of the size of their areas. Would the arrangement be different if the sectors were arranged according to the lengths of the radius?

Activity 2	Areas and Sectors

Task ► In this activity, students will fold paper sectors and investigate the relationship of the area of a sector to that of the circle.

Materials ► Compass, protractor, straightedge, paper

Procedures for Students ►

1. Draw a circle with radius approximately 5cm.
2. Fold two perpendicular diameters. How many congruent sectors are formed? What is the measure of the central angle of each sector? What is the relationship between the central angle of one sector and the central angle of one complete rotation (360°)? What is the ratio of the area of the sector to the area of the whole circle? Enter the data in the chart below.

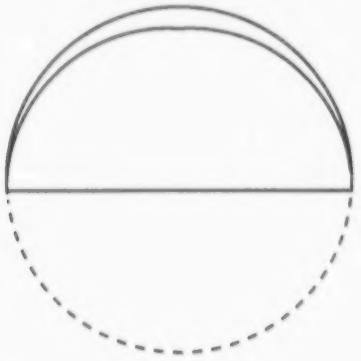

FIGURE 14

Central Angle Measure	Ratio: Central Angle to 360	Ratio of Areas: Sector to Circle

3. Fold the circle in Step 2 so that the angles are bisected. How many congruent sectors are formed? What is the measure of the central angle of each sector? What is the ratio of the area of each of these sectors to the area of the circle? Enter the data in the chart.
4. Draw a second circle and fold a diameter. Measure a central angle of 60° and fold that diameter. Fold a third diameter to bisect the obtuse central angle. How many congruent sectors are there? What is the measure of each central angle? What is the ratio of each central angle to 360°? What is the ratio of the area of one sector to the area of the circle?
5. Fold the circle in Step 4 so that the angles are bisected. How many congruent sectors are formed? What is the measure of the central angle of each sector? What is the ratio of the area of each of these sectors to the area of the circle? Enter the data in the chart.
6. Examine the data in the chart. What relationship do you see? Make a conjecture from the information in the chart.

Extension ► The same techniques can be used to make a conjecture about the relationship between the measure of the central angle and the length of an arc of a sector of a circle.

Activity 3 **Testing the Conjecture**

Task ► In this activity, students will use computers to develop a generalization about the area of a sector of a circle.

Materials ► Computer construction software, data/instruction sheet

Procedures for Students ►

1. Draw a circle P with a radius of two units. In the circle, construct a sector APB and determine the measure of the central angle. Write a ratio between the measure of the central angle and 360°. Record the data in the chart below.
2. Measure the area of the circle. Measure the area of the sector APB. Record the data in the chart below and write a ratio between the area of the sector and the area of the circle. Record the data in the chart below.

Measure of Central Angle	Ratio of area sector APB To area of circle P

3. Scale the circle to increase the radius. Repeat Steps 1 and 2 with the new circle.
4. Scale the circle to decrease the radius. Repeat Steps 1 and 2 with the new circle.
5. Repeat Steps 1, 2, 3, and 4 using different central angle measures.

Generalization ► Use all the data collected to write a generalization about the relationship between the measure of the central angle of a sector and the area of a sector of the circle. Write in your own words how you could find the area of a sector of a circle. Be sure to specify which information you would need.

ESSENTIAL ELEMENT 6: Solid Geometry

Activities for Sample Objective 78

Objective 78
Build polyhedron models using a variety of materials and investigate their properties (number of vertices, edges, and faces; shapes of faces).

Subelements
6(A)(i) and 6(A)(iv)

Activity 1	Polyhedron Property Identification

Task ► In this activity, small groups of students will identify the face shapes and the base shape and will count the number of faces and bases.

Materials ► Polyhedron models

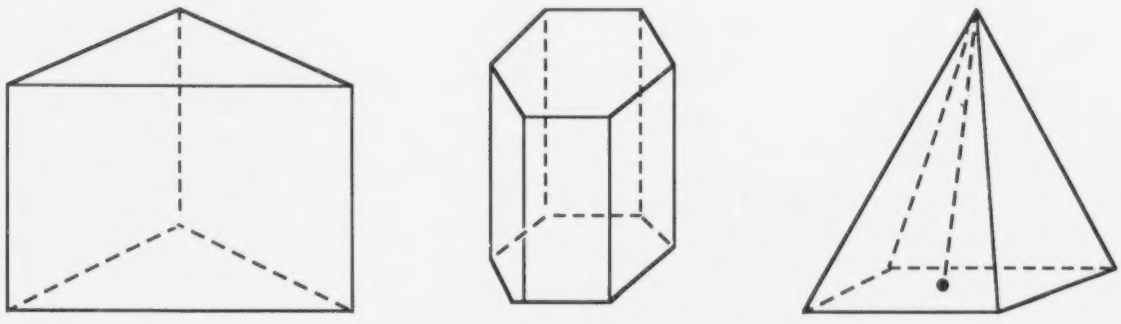

FIGURE 15

Procedures for Students ►

1. Sort the set of polyhedron models into two sets using the properties of the lateral faces, vertices, and edges, and write a description of the rule used to sort.
2. Determine the sorting rule used by each of the other groups of students.
3. Each group of students gives the rule used for sorting the set and discusses alternate rules that others may have stated.

Activity 2 **Analyzing Polyhedron Nets**

Task ► In this activity, students will group polyhedrons formed from nets according to common properties.

Materials ► Polyhedron models, nets, scissors. *(See Figure 16.)*

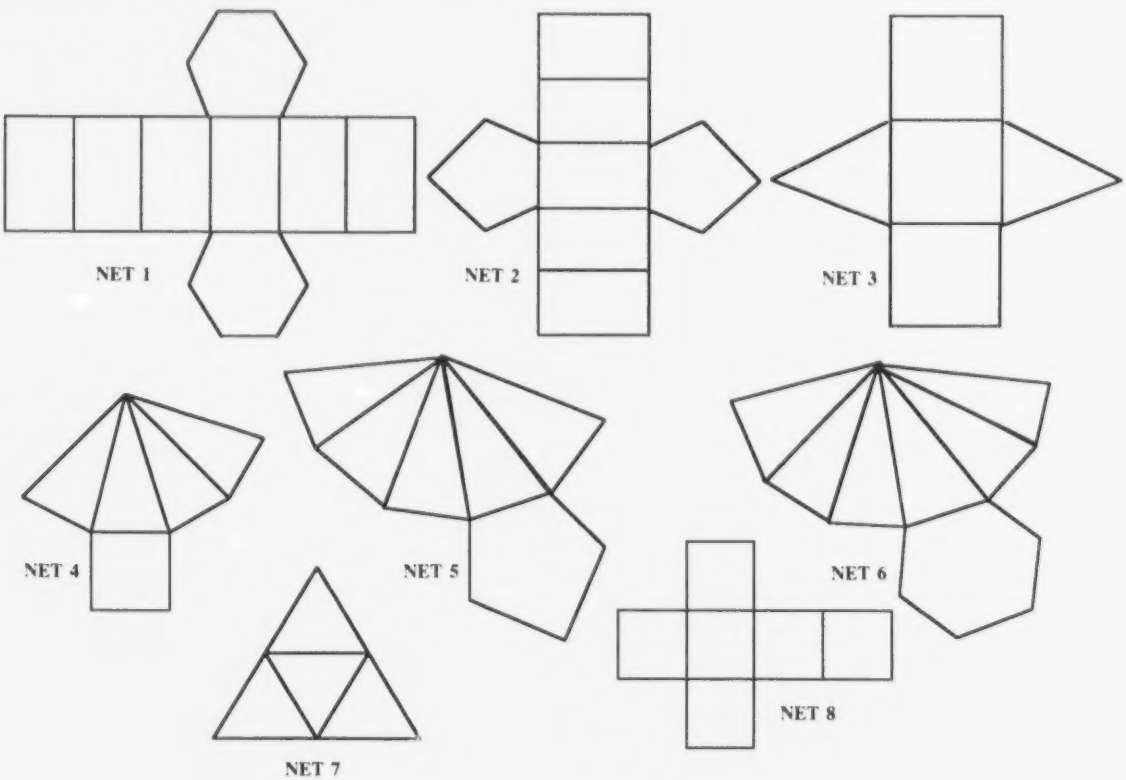

FIGURE 16

Procedures for Students ►

1. Study the nets and make a list of those that will be prisms (pyramids) when constructed.
2. Cut out each net and build the polyhedron.
3. Examine the models and complete the chart below.

Polyhedron	Number of Sides on Base	Number of Faces	Number of Vertices	Number of Edges

Investigation ►

1. Consider Net 1 and Net 6. Do you see any relationship between the number of faces, vertices, and edges?
2. Consider Net 3 and Net 5. Do you see any relationship between the number of faces, vertices, and edges?
3. Are there any other pairs for which you see similar relationships?

Activity 3	Generalizing Polyhedron Relationships

Task ► In this activity, the students will use models and the data collected to write rules for relationships between sides, vertices, and faces of polyhedron.

Materials ► Polyhedron models, chart

Procedures for Students ►

1. Reorganize the information in your chart so that all the prisms are together and the number of sides of the base are in order.
2. As the number of sides in the bases increases, what happens to the number of vertices? To the number of edges?

Generalizations ► Write as many generalizations as you can that express a relationship between and among the number of sides of the base, the number of faces, the number of vertices, and/or the number of edges. Which of these can you justify?

Teacher Information ►

Polyhedron	Number of Sides on Base	Number of Faces	Number of Vertices	Number of Edges
Triangular prism	3	5	6	9
Rectangular prism	4	6	8	12
Pentagonal prism	5	7	10	15
Hexagonal prism	6	8	12	18
Triangular pyramid	3	4	4	6
Rectangular pyramid	4	5	5	8
Pentagonal pyramid	5	6	6	10
Hexagonal pyramid	6	7	7	12

Euler's Formula: $f + v - e = 2$

Evaluation

Local Evaluation

The following material is excerpted and adapted from the National Council of Teachers of Mathematics document, *Curriculum and Evaluation Standards for School Mathematics.*

The main purpose of evaluation is to help teachers better understand what students know. Teachers can then make meaningful decisions about instruction. Evaluation includes tests, observations of students as they work, and assessments of products that are more substantial than the answer to a single question; for example, proofs, reports, and cooperative projects. Successful evaluation in Geometry will guarantee that:

1. student assessment is integral to instruction
2. multiple means of assessment are used
3. all aspects of geometry knowledge and its connections to other content are assessed
4. both classroom instruction and curriculum materials are considered equally in judging the quality of geometry instruction

Assessment must be properly aligned to the curriculum. That is, the content, processes, and skills assessed must reflect the goals, objectives, and breadth of topics specified in the curriculum. Obviously, student performance will reflect the amount of opportunity students had to learn particular information; students cannot be expected to perform well on material for which they have not received adequate instruction. But beyond this, the emphases of the assessment should reflect the emphases of instruction; questions that deal with small, relatively insignificant parts of the Geometry curriculum are not appropriate. Rather, teachers need to structure assessment items around the central ideas of Geometry, and they need to provide opportunities for students to demonstrate their understanding of the connections among the major concepts presented in the course. Too, assessment must reflect the emphasis that instruction places on technology; to the extent that calculators and computers are important during instruction, they should also be available during assessment.

Assessment in Geometry should include more than just tests. Students should also be required to write convincing arguments, both formal and informal; to construct models through drawings and the use of other manipulatives; and to solve significant problems. Such tasks are intrinsically motivating, allowing

students to integrate what they know; however, they also have extrinsic value to teachers by helping construct a more complete picture of the extent of a student's understanding. Because Geometry is a content area that is dense with concepts and principles, teachers should monitor the development of students' understanding throughout the course. When teachers notice discrepancies in student understanding early on, they can quickly remediate any significant misunderstandings indicated, so that students' future learning is not jeopardized.

One of the main goals of assessment in Geometry should be to determine the meanings that students have given to the ideas presented during instruction. Since many students lack appropriate backgrounds for the course, they will develop individual meanings, based on what they know about mathematics and the geometry of the world around them, for the ideas presented in class. Certainly not all students will construct the meaning that the teacher intended, so teachers must use assessment partly to determine the meanings their students create. Therefore, assessment needs to be flexible, in terms of the types of information gathered, modes of response, time allowed to develop a response, range of tasks involved, range of mathematics to be applied, etc.

One of the most important parts of Geometry is problem solving. Clearly, proof construction is a type of problem solving. However, many additional opportunities are available for students to engage in problem solving in Geometry; for example, making conjectures about relationships among the parts of a figure. Scoring schemes for problem-solving activities should not be "all or nothing"; rather, teachers need to give credit for progress toward the solution of a problem and for developing alternate methods for solving a problem. Problem-solving skill develops gradually, and teachers should encourage students to be persistent and to stay involved with problems while they develop their skills.

Writing and discussing content help students to develop their reasoning skills and to integrate the concepts they have learned. For example, organizing and writing a report of data generated by use of exploratory software can enable students to make conjectures through a reasoning process. In addition, oral discussions and written work help students learn that one counterexample is enough to discount the truth of a statement, while any finite number of examples is never enough to verify a general statement (at least one that applies to infinitely many cases). Students likely will not learn this by being told; rather they must gradually construct and integrate the meaning of this fact.

Assessment in Geometry needs to be broadly defined. Geometry content requires students to learn facts, concepts, and procedures and then to organize this knowledge into communications, typically in the form of informal and formal arguments or as solutions to problems. Geometry teachers have the opportunity to show students how to integrate knowledge as they learn it. The objectives suggested on pages 7-14 are consistent with this broad view of the course; teachers can support this approach through the nature of the assessment tasks they develop for students.

State Evaluation: Texas Assessment of Academic Skills

The Texas Education Agency is implementing a new statewide testing program beginning in October 1990 for the period 1990-1995. This assessment program—the Texas Assessment of Academic Skills (TAAS)—has as its primary purpose to provide Texas schools with an accurate measure of student achievement. The

new program differs significantly from TEAMS, both in content and in format. The scope of content eligible for testing has been broadened to include more of the instructional targets delineated in the essential elements, which should encourage greater emphasis on a larger set of essential elements rather than on the relatively limited set of TEAMS objectives. Every section of TAAS contains a certain number of broad objectives. These objectives remain consistent from grade to grade, representing the core concepts that form the basis for a sound instructional progression from Grade 1 through Grade 12. Instructional targets, however, differ from grade to grade. A portion of the instructional targets will be selected for assessment each year; not every instructional target will be tested every year. Thus, teachers should focus instruction on the essential elements, which reflect these targets, instead of on a narrow body of assessment objectives.

The broadened scope of the new assessment program will allow for a different focus, which will better address the academic requirements of the 1990's. Skill areas that demand little more than rote memorization will be de-emphasized, while areas that improve a student's ability to think independently and to solve problems logically will receive increased emphasis. The new assessment emphasis corresponds well to the current emphases in mathematics education, which stress the importance of teaching students higher order thinking skills.

Texas educators are required to administer TAAS to students in odd-numbered grades—3, 5, 7, 9, and 11 (exit level)—in early October beginning in 1990. The tests will assess students on objectives/instructional targets that they should have mastered in previous grades. Although the high school Geometry course goes beyond the content of the mathematics exit-level test, students who take Geometry will study many topics that are directly related to the objectives and instructional targets of TAAS. The domains, objectives, and instructional targets of the TAAS Mathematics Exit-Level Test are the following:

DOMAIN: Concepts

Objective 1	**The student will demonstrate an understanding of number concepts.**

Items for this objective assess the following instructional targets:

- Use scientific notation.
- Compare and order real numbers.
- Round whole numbers and decimals.
- Determine relationships between and among fractions, decimals, and percents.
- Find squares and square roots.

Objective 2	**The student will demonstrate an understanding of mathematical relations, functions, and other algebraic concepts.**

Items for this objective assess the following instructional targets:

- Use real number properties and inverse operations.

- Determine missing elements in patterns.
- Identify ordered pairs and solution sets in one and two dimensions.
- Apply ratio and proportion.
- Evaluate variables and expressions (formulas).
- Solve simple equations involving integers, decimals, and fractions.

Objective 3	**The student will demonstrate an understanding of geometric properties and relationships.**

Items for this objective assess the following instructional targets:

- Use the basic elements of geometry (point, line, segment, ray, angle).
- Use geometric figures and their characteristics.
- Use right-triangle properties.
- Use indirect measurement with similar triangles.
- Apply geometric properties.

Objective 4	**The student will demonstrate an understanding of measurement concepts using metric and customary units.**

Items for this objective assess the following instructional targets:

- Use metric and customary units.
- Solve problems involving measures.
- Find distance, perimeter, circumference, area, surface area, and volume.
- Recognize precision.

Objective 5	**The student will demonstrate an understanding of probability and statistics.**

Items for this objective assess the following instructional targets:

- Use counting procedures (tree diagrams, multiplication).
- Find probability of simple and compound events.
- Determine the mean, median, and the mode.

DOMAIN: Operations

Objective 6	The student will use the operation of addition to solve problems.

Items for this objective assess the following instructional target:

Use the operation of addition with real numbers in practical situations.

Objective 7	The student will use the operation of subtraction to solve problems.

Items for this objective assess the following instructional target:

Use the operation of subtraction with real numbers to solve problems.

Objective 8	The student will use the operation of multiplication to solve problems.

Items for this objective assess the following instructional target:

Use the operation of multiplication with real numbers in practical situations.

Objective 9	The student will use the operation of division to solve problems.

Items for this objective assess the following instructional target:

Use the operation of division with real numbers in practical situations.

DOMAIN: Problem Solving

Objective 10	The student will estimate solutions to a problem situation.

Items for this objective assess the following instructional target:

Estimate solutions.

Objective 11	**The student will determine solution strategies and will analyze or solve problems.**

Items for this objective assess the following instructional targets:

- Identify strategies for solving or solve proportion problems.
- Determine methods for finding or find percent and percentage.
- Determine methods for solving or solve measurement problems.
- Formulate or solve problems using geometric concepts.
- Analyze or solve probability and statistics problems.
- Make predictions.

Objective 12	**The student will express or solve problems using mathematical representation.**

Items for this objective assess the following instructional targets:

- Formulate equations/inequalities.
- Analyze or interpret graphs, charts, tables, maps, or diagrams, and use the information derived to solve problems.

Objective 13	**The student will evaluate the reasonableness of a solution to a problem situation.**

Items for this objective assess the following instructional targets:

- Determine the validity of conclusions drawn from statistical data.
- Evaluate reasonableness.

The study of high school Geometry should help students in all the content areas of the TAAS Mathematics Exit-Level Text, particularly the instructional targets under the Concepts Domain, Objectives 3 and 4, and all objectives under the Problem-Solving Domain. Geometry teachers should be aware of the TAAS objectives, but they should not emphasize these objectives to the neglect of essential elements. If the Geometry essential elements are taught as intended, students should have a solid background for the TAAS objectives.

Appendixes

Appendix A

The van Hiele Levels of Mental Development in Geometry

In the late 1950s, Pierre van Hiele and Dina van Hiele-Geldof, high school teachers in the Netherlands, conducted extensive research that dealt with the interaction of teaching and learning in geometry. The van Hieles viewed learning as a discontinuous process with jumps in the learning curve that suggested the presence of "levels." Characteristics of the five levels that the van Hieles identified are described below:

LEVEL 1: Visual Recognition

A student who is functioning at Level 1 recognizes geometric figures based on the physical appearance of the shape as a whole. The student at this level identifies, names, compares, and operates on geometric figures primarily by means of visual considerations of the concept without regard to its properties. Such a student can learn vocabulary, can identify specified shapes, and can reproduce a shape when it is shown to him or her. The student can name the parts of the figure but views these parts as wholes in and of themselves, parts that have little to do with the original figure. At this level, there is no attempt to analyze a figure in terms of its components or to generalize properties for classes of figures. The student carries out activities using trial-and-error methods that rely heavily on visual interpretation. Therefore, teachers must provide a variety of activities that give the student an opportunity to see the figures in different orientations; to find given shapes in a group of shapes or in other more complex configurations; to draw, cut, and manipulate the shapes; and to verbally describe the shapes by their appearance as a whole.

LEVEL 2: Analysis

At Level 2, students recognize geometric figures as having parts and recognize them by their parts. A student who is functioning at Level 2 reasons about geometric concepts by means of an informal analysis of component parts and attributes. Through observation and experimentation, the student begins to discern the characteristics of figures and analyzes their properties. At this point, the student is able to recognize and name properties of the figures and can compare figures using their properties. However, these properties are perceived as isolated and unrelated to each other. The student does not see the relationship between parts in a single figure, is not able to explain how certain properties of a figure are interrelated,

and does not formulate and use formal definitions. At Level 2, a student needs to be given multiple models of the geometric figures being studied, to be asked to list and verbalize the characteristics of the parts of these figures using appropriate vocabulary for components and relationships, to test relationships by telling how figures are alike and different, to sort shapes in a variety of ways according to properties, to interpret and use verbal descriptions of figures in terms of properties, and to generalize about properties of classes of figures.

LEVEL 3: Ordering

Level 3 is the point at which inductive reasoning begins and in which students organize and order properties and classifications of geometric figures. Thus, at this level, the student recognizes relationships among properties and understands the need for accurate definitions. This is the level at which the student understands logical implications and class inclusions. The student is able to give and follow informal arguments on case-specific examples. Although deductive reasoning skills are not fully developed at this level, the student can follow formal proof and use semideductive reasoning in conjunction with experimentation. Level 3 has been identified as a critical level for understanding deductive geometric proofs. However, the student functioning at Level 3 does not grasp the meaning of proof in a deductive sense, does not yet establish interrelationships among theorems, and does not fully understand the relationship between a theorem and its converse. Since logical reasoning about geometric figures is in its infancy at this level, the teacher must give the student many opportunities to explain the relationships that link the properties, to formulate and test definitions, and to give logical, informal arguments to establish one property from others, thus building a linkage among the properties.

LEVEL 4: Deduction

The student at this level uses the knowledge built in Level 3 to study geometric facts from a deductive approach. Students understand the significance of deduction as a way of establishing geometric theory within an axiomatic system, and they understand the role of undefined terms, axioms, postulates, definitions, theorems, and proof. At this level, the student can construct a proof, not just memorize it, and can compare and contrast different proofs of theorems. The student can make distinctions between a statement and its converse and can understand the role of necessary and sufficient conditions. Although formal deductive arguments can be created, the student is not concerned with the axiomatics themselves and does not compare and examine axiomatic systems for independence, consistency, or completeness. The teacher should challenge the student at this level to formulate definitions that are composed of necessary and sufficient conditions for the class of figures, to suggest alternative definitions and determine the effect on the sequence of theorems, to analyze proofs for correctness of reasoning, to compare and contrast different proofs of the same theorem, to examine the order of theorems and suggest other orderings, to state converses and contrapositives of statements and test them, and to examine the conclusions and logic of others.

LEVEL 5: Rigor

The student at this level can compare systems based on different axioms and can study various geometries in the absence of concrete models. The student can manipulate symbols without referents according to the laws of formal logic. The importance of precision in dealing with foundations and the interrelationships between structures is understood. At this level, foundations of new theories are established and new deductive systems built. The teacher should give the student opportunities to establish theorems in

different axiomatic systems; to compare axiomatic systems and explore the way in which changes in axioms affect the resulting geometry; to establish the consistency of a set of axioms; the independence of an axiom, and the equivalence of different sets of axioms; to invent generalized methods for solving classes of problems; to search for the broadest context in which a mathematical theorem or principle will apply; and to develop new insights and approaches to logical inference. This is a level of sophisticated thinking rarely reached by high school students, and it is the least addressed in research. Perhaps more students will achieve Level 5 when educators extend the van Hiele model to other areas beyond geometry and mathematics.

Appendix B

Appropriate Activities to Help Students Move From One Level to Another

To assist students in moving from a lower to a higher van Hiele level, teachers may provide activities that give students opportunities to do the following:

LEVEL 1: Visual Recognition

- use different orientations
- draw or cut shapes
- describe shapes, manipulate shapes
- find a shape in groups of shapes
- find a shape in complex configurations

LEVEL 2: Analysis

- experiment using models
- list properties
- verbally compare using correct terminology
- test relationships
- generalize about properties within classes

LEVEL 3: Ordering (Inductive)

- explain relationships among properties
- formulate and test definitions
- give logical informal arguments to link properties
- generalize about properties between and among classes

LEVEL 4: Deduction

- state converses, contrapositives, and negatives
- formulate definitions with necessary and sufficient conditions
- suggest alternative definitions and determine the effect on the system
- analyze proofs for correctness of reasoning
- compare/contrast different proofs of the same theorem
- examine the order of theorems
- examine the logic of other theorems
- examine conclusions of other theorems

Appendix C

Geometry Instructional Plan

The following instructional plan may evoke ideas from teachers to use in working with students at different levels, using different approaches.

Inquiry/Information

PURPOSE:	To ascertain the students' prior knowledge; to give students direction for further study.
TEACHER'S ROLE:	Giving information, asking questions, and giving examples and nonexamples.
METHOD:	Two-way conversation with the teacher making observations, raising questions, and introducing vocabulary and/or objects.

Guided Orientation

PURPOSE:	To provide opportunity for student exploration in carefully sequenced activities that eliminate misconception.
TEACHER'S ROLE:	Modeling and giving examples and nonexamples.
METHOD:	Short tasks directed to elicit specific, immediate responses.

Explication

PURPOSE:	To provide opportunity for student exploration in small groups or independently.
TEACHER'S ROLE:	Assisting only minimally and observing.
METHOD:	Exploration and discussion to help students express and exchange emerging views about the structures observed and to become more familiar with technical language that accompanies subject matter.

Free Orientation

PURPOSE:	To help students to begin internalization; to make the relationships between the objects of study more explicit for students.
TEACHER'S ROLE:	Observing.
METHOD:	Exploration of more complex tasks, multistep tasks, and open-ended tasks that allow students freedom of using different ways to complete the tasks.

Integration

PURPOSE:	To review and summarize with a goal of forming an overview for the new network of objects and relations.
TEACHER'S ROLE:	Facilitating closure.
METHOD:	Student discussion of properties discovered; summarization and review of origin of properties.

Appendix D

Resources

Allen, Charles. "Mission: Tangrams." *Mathematics Teacher*, February 1973.

Battista, Michael T. and Clements, Douglas H. "Using LOGO Pseudoprimitives for Geometric Investigations." *Mathematics Teacher*, March 1988.

Beckmann, Peter. *A History of Pi*. New York: St. Martin's Press, 1971.

Bell, Ellen W. and Bell, Frederick H. *Solving Geometry Mysteries Through Puzzles, Games, and Individual Activities*, Book 2. Wilkinsburg, PA: Hayes School Publishing Co., Inc., 1983.

Bezuska, Stanley; Kenney, Margaret; and Silvey, Linda. *Tessellations: The Geometry of Patterns.* Palo Alto: Creative Publications, 1977.

Bolster, L. Carey and Maletsky, Evan M., eds. "Tangram Mathematics." *Mathematics Teacher*, February 1977.

Burns, Marilyn. *The Book of Think.* Boston: Little, Brown and Company, 1976.

_____. *The Math Solution-Teaching for Mastery Through Problem Solving.* Sacramento: Marilyn Burns Education Association, Inc., 1984.

Clements, Stanley; O'Daffer, Phares; Cooney, Thomas; and Dossey, John. *Geometry.* Menlo Park, CA: Addison-Wesley Publishing Company, 1990.*

Cox, Philip L., *Geometry, an Informal Approach.* Newton, MA: Allyn & Bacon, Inc., 1986.

Coxford, Arthur and Usiskin, Zalman. *UCSMP Geometry.* Glenview, IL: Scott, Foresman and Co., 1990.

Davidson, Patricia S. and Willcutt, Robert E. *Spatial Problem Solving with Paper Folding and Cutting.* New Rochell: Cuisenaire Co. of America, 1984.

Easterday, Kenneth E.; Henry, Loren L.; and Morgan, Simpson F. *Activities for Junior High School and Middle School Mathematics.* Reston, VA: National Council of Teachers of Mathematics, 1986.

Ernst, Bruno. *Adventures With Impossible Figures.* Norfolk, England: Tarquin Publications, 1986.

Ewbank, William. "If Pythagoras Had a Geoboard." *Mathematics Teacher*, March 1973.

Farrell, Margaret A. *Geoboard Geometry.* Palo Alto: Creative Publications, 1971.

_____. *Imaginative Ideas for the Teacher of Mathematics, Grades K-12.* Reston, VA: National Council of Teachers of Mathematics, 1988.

Foerster, Paul A. *Algebra and Trigonometry, Functions and Applications.* Menlo Park, CA: Addison-Wesley, 1984.

Fraser, Sherry (project director). *SPACES: Solving Problems of Access to Careers in Engineering and Science.* Palo Alto: Dale Seymour Publications, 1982.

*State-adopted textbook (1990-1996).

Fuys, D.; Geddes, D.; and Tischler, R. "The van Hiele Model of Thinking in Geometry Among Adolescents." *Journal for Research in Mathematics Education, Monograph 3,* 1988.

Greening, Jay. *Geometry, a Blended Approach.* Columbus, OH: Charles E. Merrill, 1989.

Hirsch, Christian; Schoen, Harold; Samide, Andrew; Coblentz, Dwight; and Norton, Mary Ann. *Activities for Implementing Curricular Themes.* Reston, VA: National Council of Teachers of Mathematics, 1990.

_____. *Geometry.* Glenview, IL: Scott, Foresman and Co., 1984.*

Jacobs, Harold R. *Geometry.* New York: W. H. Freeman and Co., 1987.

Jurgensen, Ray; Brown, Richard; and Jurgensen, John. *Geometry.* Boston: Houghton, Mifflin Company, 1990.*

Kalin, Robert and Corbitt, Mary Kay. *Geometry.* Englewood Cliffs, NJ: Prentice Hall, 1990.*

Keedy, Mervin; Bittinger, Marvin L.; Smith, Stanley A.; and Nelson, Charles W. *Informal Geometry.* Menlo Park, CA: Addison-Wesley, 1987.

Kennedy, Joe and Thomas, Diane. *Kaleidscope Math.* Paló Alto, CA: Creative Publications, 1989.

Kespohl, Ruch C. *Geometry Problems My Students Have Written.* Reston, VA: National Council of Teachers of Mathematics, 1979.

Kidd, Kenneth P.; Myers, Shirley S.; and Cilley, David. *The Laboratory Approach to Mathematics.* Chicago: Science Research Associates, 1970.

Lepowsky, William. "The Total Angular Deficiency of Polyhedra." *The Mathematics Teacher,* December 1973.

Lindquist, M.M. and Shulte, A. P., eds. *Learning and Teaching Geometry, K-12; 1988 Yearbook.* Reston, VA: National Council of Teachers of Mathematics, 1988.

Lindquist, Mary Montgomery, ed. *Learning and Teaching Geometry, K-12: The 1987 NCTM Yearbook.* Reston, VA: National Council of Teachers of Mathematics, 1987.

Litwiller, Bonnie H. and Duncan, David R. *Activities for Maintenance of Computational Skills and Discovery of Patterns.* Reston, VA: National Council of Teachers of Mathematics, 1980.

Lund, Charles. *Dot Paper Geometry.* New Rochelle: Cuisenaire Company of America, Inc., 1980.

Lyng, Merwin J. *Dancing Curves.* Reston, VA: National Council of Teachers of Mathematics, 1978.

*State-adopted textbook (1990-1996).

Maletsky, Evan M. and Hirsch, Christian R., eds. *Activities from the Mathematics Teacher.* Reston, VA: National Council of Teachers of Mathematics, 1981.

Moore, Margaret L. *Geometry Problems for LOGO Discoveries-Explorations in Turtle Geometry.* Palo Alto: Creative Publications, 1984.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics, 1989.

_____. *Mathematics Teacher-Special Issue: Geometry.* Reston, VA: National Council of Teachers of Mathematics, September 1985.

Olson, Alton T. *Mathematics Through Paper Folding.* Reston, VA: National Council of Teachers of Mathematics, 1975.

Pederson, Katherine. *Trivia Math: Advanced Topics.* Palo Alto: Creative Publications, 1988.

_____. *Trivia Math: Geometry.* Palo Alto: Creative Publications, 1987.

Picciotto, Henri. *Pentomino Activities.* Palo Alto: Creative Publications, 1986.

_____. *Supertangram Activities.* Palo Alto: Creative Publications, 1986.

Posamentier, Alfred S. and Sheridan, Gordon. *Math Motivators! Investigations in Geometry.* Menlo Park, CA: Addison-Wesley, 1985.

Ranucci, E. *Seeing Shapes.* Palo Alto: Creative Publications, 1973.

Roberge, James. "Tangram Geometry." *The Mathematics Teacher,* March 1977.

Roe, T. Sundara. *Geometric Exercises in Paper Folding.* New York: Dover Publications, Inc., 1966.

Schadler, Reuben. *Geometry Problems—One Step Beyond.* Palo Alto: Dale Seymour Publications, 1984.

Schwartz, J. and Yerushalmy, M. *The Geometric Supposer: Circles* [software]. Pleasantville, NY: Sunburst, 1986.

_____. *The Geometric Supposer: Quadrilaterals* [software]. Pleasantville, NY: Sunburst, 1985.

_____. *The Geometric Supposer: Triangles* [software]. Pleasantville, NY: Sunburst, 1985.

Serra, Michael. *Discovering Geometry.* Key Curriculum Press, 1989.

Seymour, Dale and Schadler, Reuben. *Creative Constructions.* Palo Alto: Creative Publications, 1974.

_____. *Tangramath.* Palo Alto: Creative Publications, 1971.

Sobel, Max A., ed. *Readings for Enrichment in Secondary School Mathematics.* Reston, VA: National Council of Teachers of Mathematics, 1988.

Sobel, Max A. and Maletsky, Evan M. *Teaching Mathematics*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1975.

Stokes, William T. *Gems of Geometry.* Palo Alto: William T. Stokes, Publisher, 1978.

van Hiele, P. M. *Structure and Insight: A Theory of Mathematics Education.* New York: Academic Press, 1986.

Will, Herbert III. *Leonardo's Dessert, No Pi.* Reston, VA: National Council of Teachers of Mathematics, 1985.

Williams, Wayne. *Quizzles.* Palo Alto: Dale Seymour Publications, 1982.

Winter, M. J.; Lappan, G.; Phillips, E.; and Fitzgerald, W. *Middle Grades Mathematics Project: Spatial Visualization*, 1986.

Wise, Alan. *Spotlight on Mathematics.* New York: Random House, Inc., 1981.

ACKNOWLEDGMENTS

Texas Education Agency mathematics staff members are producing high school curriculum documents such as this one with the help of committees of educators from different geographic regions. Members of a committee from the Houston area were the major contributors to the development of *Guidelines for Teaching Geometry*. The committee consisted of George Bright, University of Houston at University Park; Maggie Dement, Spring Branch ISD; Eva Gates and Kathleen Murrell, Pasadena ISD; Betty Hall and Madolyn Reed, Houston ISD; Alma Mabry, Deer Park ISD; Judy Mattair and Mary Ann Norton, Spring ISD; and Bonnie McNemar, Harris County Department of Education. In addition, numerous other mathematics educators, representing school districts, education service centers, and institutions of higher education across the state, reviewed drafts and offered many useful suggestions.

COMPLIANCE STATEMENT

TITLE VI, CIVIL RIGHTS ACT OF 1964; THE MODIFIED COURT ORDER, CIVIL ACTION 5281, FEDERAL DISTRICT COURT, EASTERN DISTRICT OF TEXAS, TYLER DIVISION

Reviews of local education agencies pertaining to compliance with Title VI Civil Rights Act of 1964 and with specific requirements of the Modified Court Order, Civil Action No. 5281, Federal District Court, Eastern District of Texas, Tyler Division are conducted periodically by staff representatives of the Texas Education Agency. These reviews cover at least the following policies and practices:

(1) acceptance policies on student transfers from other school districts;

(2) operation of school bus routes or runs on a non-segregated basis;

(3) nondiscrimination in extracurricular activities and the use of school facilities;

(4) nondiscriminatory practices in the hiring, assigning, promoting, paying, demoting, reassigning, or dismissing of faculty and staff members who work with children;

(5) enrollment and assignment of students without discrimination on the basis of race, color, or national origin;

(6) nondiscriminatory practices relating to the use of a student's first language; and

(7) evidence of published procedures for hearing complaints and grievances.

In addition to conducting reviews, the Texas Education Agency staff representatives check complaints of discrimination made by a citizen or citizens residing in a school district where it is alleged discriminatory practices have occurred or are occurring.

Where a violation of Title VI of the Civil Rights Act is found, the findings are reported to the Office for Civil Rights, U.S. Department of Education.

If there is a direct violation of the Court Order in Civil Action No. 5281 that cannot be cleared through negotiation, the sanctions required by the Court Order are applied.

TITLE VII, CIVIL RIGHTS ACT OF 1964; EXECUTIVE ORDERS 11246 AND 11375; TITLE IX, 1973 EDUCATION AMENDMENTS; REHABILITATION ACT OF 1973 AS AMENDED; 1974 AMENDMENTS TO THE WAGE-HOUR LAW EXPANDING THE AGE DISCRIMINATION IN EMPLOYMENT ACT OF 1967; AND VIETNAM ERA VETERANS READJUSTMENT ASSISTANCE ACT OF 1972 AS AMENDED IN 1974.

It is the policy of the Texas Education Agency to comply fully with the nondiscrimination provisions of all federal and state laws and regulations by assuring that no person shall be excluded from consideration for recruitment, selection, appointment, training, promotion, retention, or any other personnel action, or be denied any benefits or participation in any programs or activities which it operates on the grounds of race, religion, color, national origin, sex, handicap, age, or veteran status (except where age, sex, or handicap constitute a bona fide occupational qualification necessary to proper and efficient administration). The Texas Education Agency makes positive efforts to employ and advance in employment all protected groups.